An Optimal and Automatic Graph Cut Method for Biomedical Images Using Compactness Measure

Sonia Nardotto, Laura Gemme, and Silvana G. Dellepiane

***Abstract*—This work aims to achieve an automatic and optimal graph cut phase based on a segmentation method presented in a previous paper. A graph-based segmentation algorithm, starting from a seed point belonging to the region of interest (ROI), is able to find the Minimum Path Spanning Tree (MPST) by using a new cost function and an optimal aggregation criterion. In order to extract the ROI, a graph-cut of the obtained tree is absolutely necessary.**

By definition, the main drawback of the graph-based segmentation methods is the loss of spatial and contextual information. To overcome this problem, a new method based on compactness measure is here proposed

The present approach is applied to the biomedical field, considering Magnetic Resonance Imaging (MRI) volumes of the hand and neurological districts.

***Index Terms*—Automatic segmentation, graph-based segmentation, graph cut, compactness measure.**

I. Introduction

Image segmentation has an important role in the image processing framework. Its aim is to partition an image into several non-overlapped regions whose union is the entire image or, in the case of Region of Interest (ROI) segmentation, the result is a bi-partition of the image in two regions, usually called “object” and “background”.

Depending on the application, the purpose of the segmentation becomes specialized to detect parts of the image, which are meaningful for the domain considered.

This topic has been widely studied in the past years and has still a great interest since fully automated methods are difficult to achieve and scarcely reported in the literature. Several papers have been published on image segmentation, the most recent dealing with watershed and energy functions [1]. In the latter category, the graph-based segmentation methods are included, as the one here described. With regard to that, in the following section some innovative aspects are proposed as compared with the existing approaches.

In fact, our aim is to optimize and make automatic the graph cut (GC) phase of the new segmentation method presented in [2]. This new method envisions the formulation of a new cost function, based on a known algorithm [3] usually applied to telecommunication networks problems. This algorithm, although commonly known, is not usually considered and employed in the context of graph-based segmentation methods. The Minimum Path Spanning Tree (MPST) is generated according to the above newly defined cost function, which minimizes the largest vertex weight in the path and is equivalent to the maximum capacity path problem.

Such a segmentation is an unsupervised adaptive method, independent of any model, parameter, threshold, or order of analysis.

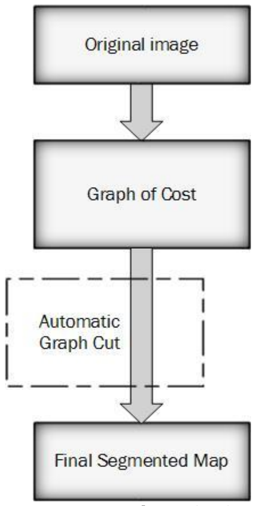

Fig. 1. The automatic graph-cut (GC) method.

In order to extract the region of interest a final graph cut step is necessary. Generally, most of the graph cut methods are based on the optimization of an energy function that considers the sum of the weight edges between the two sets to be partitioned. Here a different method is suggested just to automate this phase.

As it is generally known, the graph-based approach is not able to take into account spatial or morphological information, which is indeed extremely useful in the analysis of meaningful borders and regularized shapes. The loss of contextual information means that local contrasts and edges are not appropriately considered.

In order to overcome these problems, we propose a method to determine the value of the cut based on a classical compactness measure. The use of this criterion is due to the quite regular shape and compactness of the objects that we want to identify (the wrist bones and neurological tissues).

The compactness measure is given by the ratio between the square of perimeter and the area of the object and it is computed for each possible cut value that ranges from 0 to the maximum cost value that can be represented. In this way, the compactness is a function of the cut value. The optimal choice of the cut is given by the value corresponding to the first local minimum.

Fig. 1 shows the main steps of the proposed method.

The rest of this paper is structured as follows: in the following section, a brief description of graph theoretic techniques is reported, with a major explanation of the graph

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cut concept. In Section III, the graph-based segmentation method is recalled and the proposed automatic approach for the choice of the cut value is described. Applications and discussion of the results are presented in Section IV.

II. State of the Art

In this section, a brief review of the graph-based segmentation techniques is presented, followed by an in-depth description of the graph cut. Usually, these methods represent the problem in terms of a graph $G(V,E)$, where V, the set of the vertices, corresponds to the spels in the image, and E, the set of the edges, represents pairs of neighboring spels. A weight is associated with each edge according to specific criteria, like the intensity value. These techniques, following the graph theory, can be grouped into five main categories [4].

1) Minimal Spanning tree (MST) based methods

This concept of the graph theory is used in image segmentation considering the relationship between the MST and the cluster structure. In particular, the approach consists in minimizing the sum of weight edges among all spanning trees. In the literature, different algorithms exist for obtaining the minimum spanning tree (such as Prim's Algorithm, Kruskal's Algorithm) [5]. To obtain the segmentation, a partition of the graph into different sub-graphs is performed by removing edges.

2) Graph Cut with Cost Function

Graph cut is used to partition a graph into meaningful regions considering different cost functions. The most common concept of cut is the sum of weights on edges linking two different components. In addition to the classical definition, other cost functions are considered in the context of normalized cut methods: Normalized Cut, Region Cut, Mean Cut and Ratio Cut [4]. Generally, this problem is solved with optimization methods, minimizing the cut value.

3) Graph Cut on Markov Random field (MRF)Models

This method combines the contextual information such as image pixel and features provided by MRF with the graph cut theory. Similar to the previous approach, the MRF framework formulates a problem in terms of minimizing an energy function.

4) The Shortest Path based Methods

A path problem consists in finding an optimal path between two vertices. The possible variants of a path are single source-single sink (s-t graph), single source, and all pairs [6]. There are many measures for path optimality [7]. One is the minimization of weight edge sum over all paths between the nodes. Another one is the minimum edge on the path (the so-called bottleneck). Maximizing this measure defines the maximum bottleneck path problem. The last measure is the minimization of a non-decreasing sequence.

5) Other methods can be Random walker and Dominant set based method.

Graph Cut in Image Segmentation

In this section some concepts of GC are recalled [1], [4], [7]. Let $G(V,E)$ be an undirected weighted graph. The set of the vertices V is composed of two types of nodes: the neighborhood nodes, which correspond to the spels, and the terminal nodes, which consist of s (source) and t (sink). This graph is also called *s-t* graph. Usually, in image applications, s and t represent the object and the background respectively. In addition, the set of the edges E is divided into two groups. The edges called *n*-links connect the neighboring spels within the image; the edges called *t*-links connect the terminal nodes with the neighborhood nodes.

A cut C is a subset of edges connecting the two partitions in which the graph is divided, i.e., the object and the background.

The cost of the cut is usually the sum of the weights on edges, as given by this expression [1]:

$$C = \sum_{e \in C} w_e \tag{1}$$

where w_e is the weight of the edges belonging to the cut.

The minimum cut is the one which minimizes the sum in Equation (1).

In order to obtain a precise and accurate segmentation, the cut should occur at the boundary between object and background. Within this context, the energy function is defined as follows [4]:

$$E(A) = \alpha R(A) + B(A) \tag{2}$$

where A is the vector that defines the segmentation, R is the regional term; B is the boundary term and α is the relative importance factor between the two previous terms.

In the context of graph cut, a generalization of the minimal cut problem is the minimal multi-terminal cut problem [8]. Given a weighted graph and a subset of nodes, called terminals, the problem is to find a minimum weight set of edges that separates each terminal from all the others. The result is to find as many partitions as the number of the terminals. In the case of two terminals, the problem reduces to the *s-t* cut [9].

The method here proposed is a new approach, which falls into the first and the fourth group. In fact, the algorithm finds the Minimum Path Spanning Tree, which is a minimum spanning tree with respect to the bottleneck function. It is a new algorithm because the MST does not consider the sum of weight edges, but the minimum of the largest weight edge over the path.

III. Proposed Method

This work, referring to the 3D-segmentation method described in [2], focuses on automatizing and optimizing the graph cut phase. The volume is mapped into a graph, where each voxel corresponds to a vertex and pairs of neighboring vertices represent the edges. The objective is to extract different regions of interest from the original volume. This algorithm, starting from a single seed belonging to the ROI, proceeds with an optimal growing criterion. As final step, a graph cut is computed. Differently from the classical graph-cut approaches, which minimize cost functions given by the sum of weight edges, here a method based on compactness measure is proposed.

A. The Segmentation

In this section, the main steps of the segmentation algorithm are described. The considered method is a single

source algorithm that applies a graph-based segmentation driven by research into the minimum cost paths for the analysis of digital volumes [2].
The volume is an undirected, vertex weighted, grid graph.

As a preliminary step, with respect to the seed point s, the algorithm assigns the weight to each vertex representing the dissimilarity with the seed and given by this formula:

$$\forall v_i \in V, \qquad w_s(v_i) = |I(v_i) - I(s)| \tag{3}$$

where I is the intensity map and v_i is a generic vertex.

Then, an optimal growing mechanism starts to obtain a spanning tree that is minimum with respect to the cost function [2]:

$$f_{w_s}(v_i) = \min_{\pi(s,v_i)}[\max_{x\in\pi(s,v_i)} w_s\,(x)] \tag{4}$$

where $\pi(s, v_i)$ is the path from the seed s to a generic vertex v_i. This function is the formulation of bottleneck single source shortest path [3], which, although commonly known, is not usually considered and employed in the context of graph-based segmentation methods.

This aggregation process decides which nodes are the candidate ones to be analyzed (based on the already analyzed nodes) and selects the best path. This process continues until the entire volume is analyzed. The obtained paths set up a Minimum Path Spanning Tree.

Compared to other existing segmentation methods, a new cost function is here proposed. It allows the process to be adaptive to both local and global context, to be optimal and independent from the order of analysis, requiring a single iteration step. The new cost function and the propagation algorithm are the main innovative aspects of this method.

Finally, a graph cut phase is applied to extract the ROI.

In previous works finalized to the study of the efficiency of the method in separating single wrist bones, this value is chosen in a semi-interactive way, corresponding to the maximum graph cut that does not generate leakage problems.

Each value of the cost function corresponds to a different level. The cut is applied when there is a large difference ("gap") between the orders of two consecutive levels. This is due to the presence of local contrast or edges. When a graph cut is performed according to a gap, it is assured that the region is made of similar voxels. Such a result can be interpreted as a multi-level segmentation: with a cut just before the gap, we obtain the region of interest. After the gap, there can be an enlargement of the region until the entire image is covered.

It is proved that a significant gap appears for ROI with homogeneous grey levels or in the case of bridges between objects.

In this work, we suggest a method to automate this phase. The morphological information, like shape and compactness measures, lost in the graph-based approaches, is here considered.

B. The Compactness Approach

A possible way to achieve an automatic cut detection is to exploit some morphological features, such as perimeter, area, compactness, and others [10]. Perimeter is a simple geometric measure that can be calculated from the contour of the object.

The ROI area is a direct counting of the voxels number. The geometric measurements of perimeter and area are both insignificant when an object is observed at different spatial resolution scales. In this case, you need to define features which are invariant to scale.

The well-known shape-factor, C, is a simple compactness measure (adimensional) defined as:

$$C = \frac{P^2}{A} \tag{5}$$

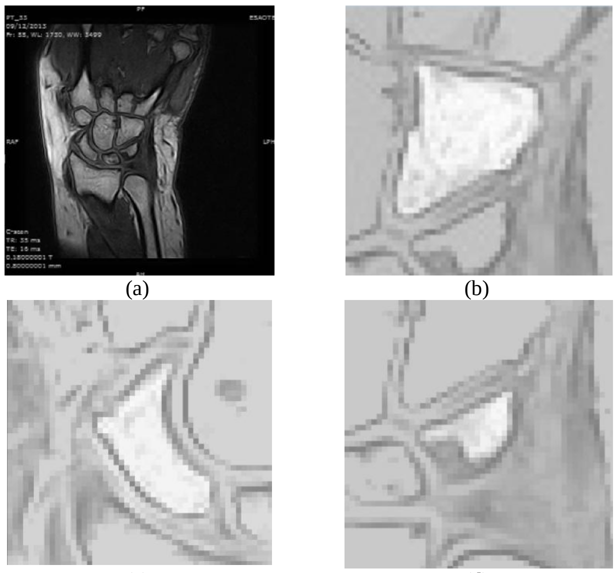
where P and A are respectively the perimeter and area of the region.

The circle is the most compact geometrical shape in Euclidean space with a minimum value of compactness equal to 4π. For square shape compactness is 16, and for an equilateral triangle is $36/\sqrt{3}$. Area, perimeter, and compactness are invariant measures with respect to rotation of the object.

In this work, the compactness information is used to find an automatic cut value for different type of images, independently of noise and resolution.
In the figures of the following section the trend of perimeter, area and compactness are drawn as a function of the cut value, where the best threshold is pointed out. As one can see, the best value of cut is exactly in the local minimum of compactness, given by equation (5). This is evident by looking the binary images, where the best automatic segmented image is characterized by a regular border and holes filled. On the contrary, if we consider the image where cut value is too high, we can see that the value of perimeter also is too high as compared to the area value.

Currently the system is able to detect automatically the best cut value and give to the user the Final Thresholded Map.

IV. RESULTS



(a) (b) (c) (d)

Fig. 2. (a) A coronal slice of the patient33 original volume (b) Hamate Bone negative cost image, (c) Scaphoid Bone, (d) Triquetral Bone.

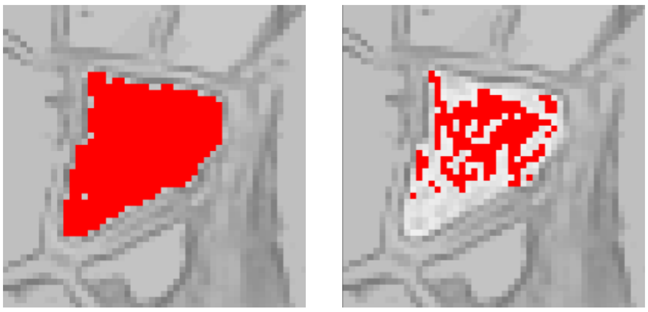
The proposed method has been applied in the biomedical domain, considering the extraction of wrist bones and the analysis of neurological cases from real Magnetic Resonance volumes. The experiments related to three patients for wrist bones and 1 patient for the brain district for a total of 11 volumes are reported and discussed as follows.

The database (DB) used for wrist bones is described in paper [11] and is made of 100 MRI T1-weighted volumes acquired by the 0.2 Tesla ARTOSCAN (Esaote Spa, Genoa, Italy). Each volume is made of approximately 120 slices of size 256×256 pixels. The quality of this dataset in terms of resolution, contrast, and noise is very low. In fact, for example, the cortical bone is large approximately 1 voxel and it is difficult to see also for the user.

The BrainWeb DB [12] contains simulated brain volumes; the volume size is $181\times217\times181$, voxel size being 1 mm^3, corrupted by 0% noise level, with 0% RF.

For each volume of the hand, starting from a seed point decided by the user, the wrist bones are extracted separately.

Fig. 2(a) shows one coronal slice of the original wrist volume of patient 33. Fig. 2(b), Fig. 2(c), Fig. 2(d), represent a (50×50) cropped image of the negative cost value as found through the proposed method for the three hamate, scaphoid, and triquetral bones. The brightest voxels, corresponding to the lowest cost values, most likely belong to the searched ROI.



(a) (b)

Fig. 3. Hamate33. (a) In red automatic GC result, (b) Example of wrong GC result.

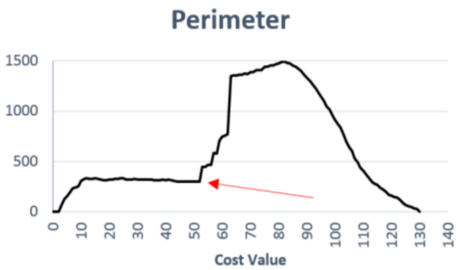


Fig. 4. Perimeter value as a function of GC cost value for Hamate33.

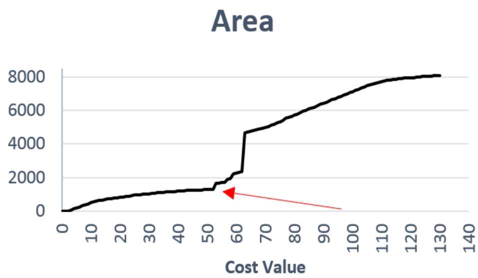


Fig. 5. Area value as a function of GC cost value for Hamate33.

The graph cut results in a thresholding of the whole cost volume. In Fig. 3 two segmentation results are shown for hamate bone according to two GC values. A good result is displayed on the left side (GC value equal to 52), while on the right side, an example of bad graph cut value choice can be seen. As expected, since the red ROI in Fig. 3(b) is a wrong segmentation of the bone, its perimeter length in relation to its area is much higher than the one measured for ROI in Fig. 3(a), where a compact red ROI is shown, whose shape factor (Equation (5)) is minimized.

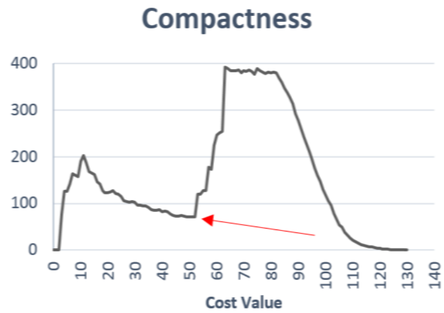
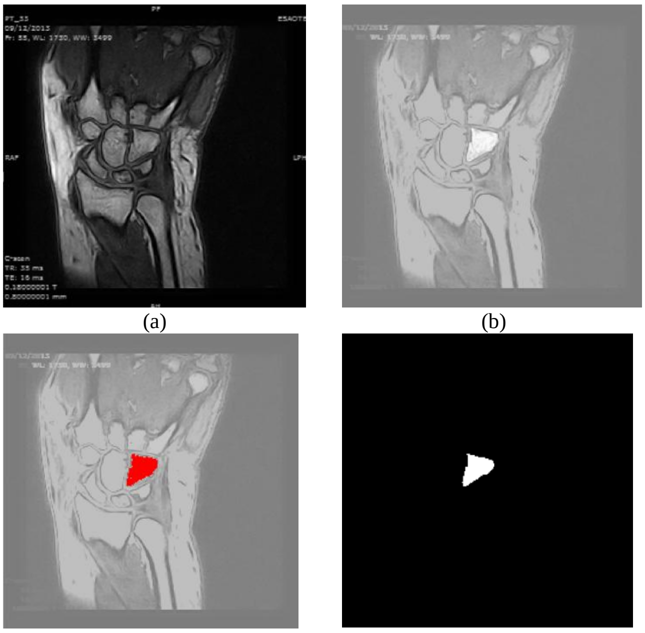


Fig. 6. Compactness value as a function of GC cost value for Hamate33.



(a) (b) (c) (d)

Fig. 7. (a) A coronal slice of the patient 33 original volume , (b) Cost value image for Hamate Bone, (c) Automatic GC, (d) Segmented ROI for Hamate Bone.

Fig. 4, Fig. 5, and Fig. 6 report the values of perimeter, area, and compactness for hamate bone of patient 33 as a function of GC value. As one can see, the graph cut value found in correspondence of the significant local minimum of compactness is equal to 52, corresponding to a good ROI for the searched bone.

In Fig. 7 the processing phases starting from the original slice are shown without cropping for hamate bone.

TABLE I: BEST GRAPH CUT VALUE COMPARED WITH AUTOMATIC VALUE

	Best GC	Automatic GC	Error (difference)
Hamate3	183	185	2
Scaphoid3	207	207	0
Triquetral3	183	176	7
Hamate17	208	210	2
Scaphoid17	210	212	2
Triquetral17	199	201	2
Hamate33	202	204	2
Scaphoid33	231	226	5
Triquetral33	202	204	2

Hamate

Scaphoid

Triquetral

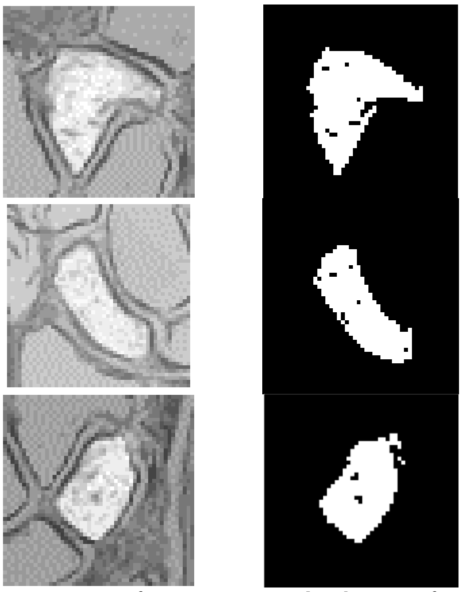


Fig. 8. Result of segmentation of patient 3 compared with Image of Cost values (cropped).

Hamate

Scaphoid

Triquetral

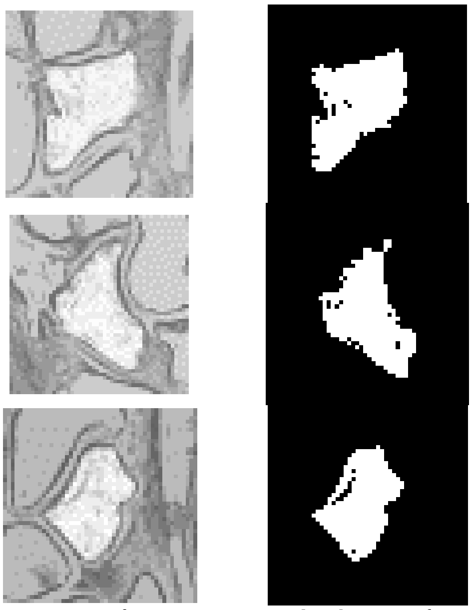


Fig. 9. Result of segmentation of Patient 17 compared with Image of Cost values (cropped).

Fig. 8, Fig. 9 and Fig. 10 show the result for hamate, scaphoid and triquetral bones for patients 3, 17, and 33 respectively.

According to the medical segmentation of the given DB, the Best Graph Cut value has been computed for each bone. Such values, as compared with the automatic value found by the automatic GC value choice are reported in Table I. One can observe how the difference between the best and the automatic values are always very low, ranging from 2 to 7.

However, since the segmentation proposed is robust with respect to small changes in the GC value, even the largest error in the parameter value does not affect the ROI result in a meaningful way. Such a fact can be observed by looking at Fig. 11, which shows the segmented triquetral bone as extracted with the best value and the automatic value. Differences between the two results are extremely low.

For some bone more affected by noise or low contrast, the best cut might be slightly shifted with respect to the found minimum. This condition does not compromise the final correct result of segmentation.

Hamate

Scaphoid

Triquetral

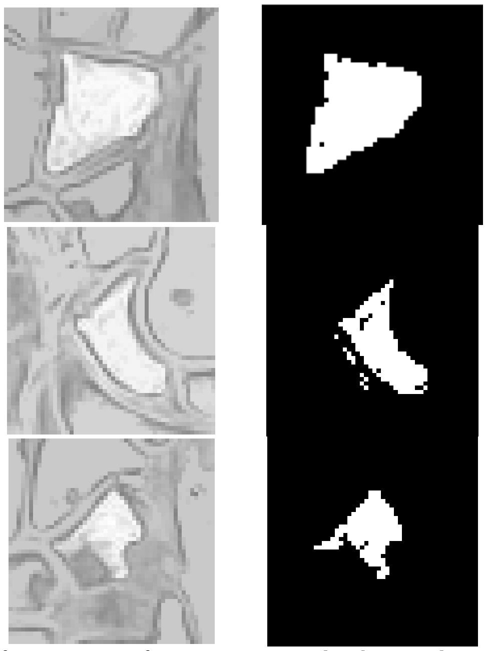
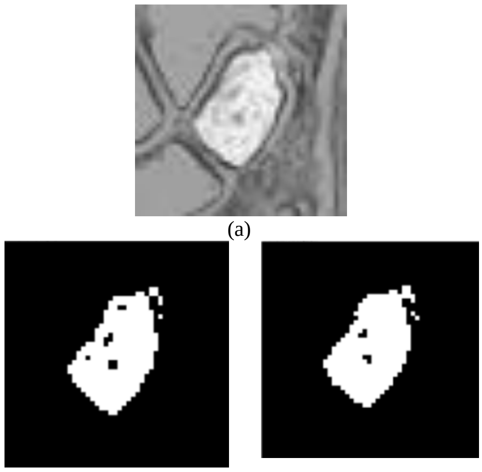


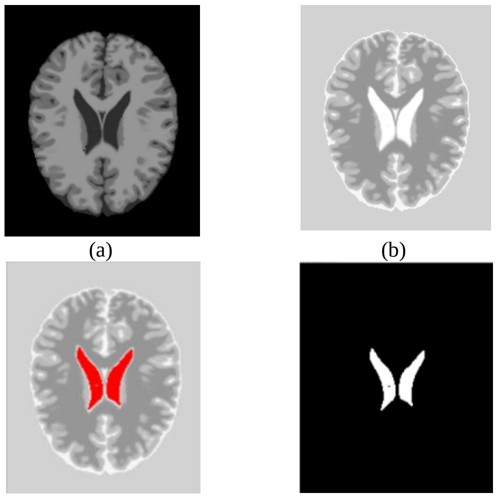
Fig. 10. Result of segmentation of patient 33 compared with cost-value image (cropped).



(a)

(b) (c)

Fig. 11. Triquetral3 Bone. (a) Cropped image of cost values, (b) Segmented image with best GC, (c) Segmented image with automatic GC.



(a) (b)

(c) (d)

Fig. 12. (a) Original axial brain slice from MRI, (b) Cost function image for ventricles, (c) Automatic GC, (d) Segmented ventricular ROI slice.

Similar or even better results can be obtained when the method is applied to an MRI volume of the brain district, as shown in Fig. 12. When choosing the GC value as the minimum value of connectedness function of Fig. 13, the ventricular system is correctly segmented in an automatic way, as shown in Fig. 12(d).

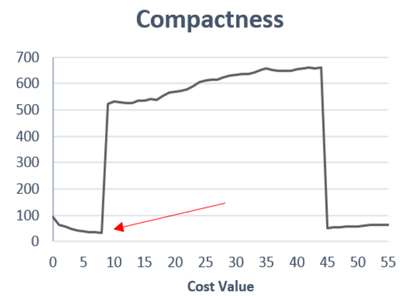


Fig. 13. Graph of compactness value for brain.

V. Conclusions

This paper aims to automatize and optimize the graph cut phase of the graph-based segmentation method proposed in a previous work. The algorithm, starting from a seed point belonging to the ROI, finds the Minimum Path Spanning Tree with respect to a new cost function. As a final step, in order to locate and detect the region of interest, a graph cut phase is performed. Generally, the graph cut methods optimize an energy function but due to the graph conformation, the spatial and contextual information are lost. For these reasons, the proposed approach is based on spatial features and the cut value is chosen by minimizing the compactness function. The experiments are applied to MRI volumes of the wrist and the brain in order to isolate the carpal bones and the ventricles, thus proving the flexibility and adaptation of this method to different anatomical districts and organs of interest.

As one can see, the segmented images obtained through the automatic method are very similar to the manual method.

This demonstrates that the Automatic GC is a very simple automatic method, able to strongly reduce the user's intervention.

As future work this graph cut approach will be applied to all the bones of the patients of the DB, in order to make it more robust and to perform more evaluations.

Acknowledgments

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