Assessment and Allocation of Wheeling Service Prices among System Users through Power Tracing

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Abstract—**Wheeling service pricing has become a very crucial issue in the emerging competitive power market scenario, because pricing has become the central phenomenon in any commodity trading. This paper establishes the importance of power flow tracing for fair allocation of wheeling prices among wheeling system users, by comparing two tracing techniques. Power tracing is necessary tool for determining the contribution of sellers and buyers in a specific wheeling transaction under pool power market environment. For evaluating wheeling facility usage by sellers and buyers, in pool power market, two power tracing techniques i.e. Rudnick tracing and Bialek tracing have been implemented. The tariff required for wheeling services, should be sensitive to distance, direction and the magnitude of power wheeled and must be fair enough for users as well as for owners. To acquire these characteristics in the present day wheeling pricing, the usage-based wheeling pricing techniques are gaining popularity. In the present paper, original apparent flow-mile technique and six usages based apparent flow-mile techniques, have been discussed to examine the limitations and feasibility of above said tracing techniques. The present approach of assessment and allocation of wheeling prices through tracing has been examined and analyzed for IEEE 30-bus system based pool power market. The results so obtained, significantly establishes the limitations and feasibility for the two above said tracing techniques for given topological system configuration.**

Index Terms—**Power tracing, wheeling pricing, proportional sharing principle, generalized distribution factors, AF-mile, sellers and buyers.**

I. INTRODUCTION

The recent trend of unbundling in Electric Supply Industry (ESI) stimulated a renewed interest in pricing power wheeling services. Because in this new era of trading, all the market participants (sellers, buyers, and wheelers) need to know the cost for wheeling services to make correct economic decisions on the various types of facilities, they should promote or curtail. The sellers and buyers are required to know such prices in order to make efficient use of wheeling facilities and to make better profit margins through wheeling. At the same time Wheelers (wheeling service providers or owners) need to know these prices to make right economic and technical decisions for upgrading the existing facilities and to ensure optimized utilization of the existing facilities.

With the deregulation in the utility industry, customers have the option to purchase services and energy from different sources to obtain better quality and price. Since it is impossible to colour electrons, there is no standard way to estimate correct wheeling path and wheeling prices within the utility industry [1].

For making business transactions, in the competitive electric power market, two options are there [2]. First is direct access, means customer would negotiate directly with suppliers for purchasing electricity. Other option is centralized market place (known as "pool"). The former one provides better choice but there is no provision for system security responsibilities and wheeling price assessment. On the other hand, a pool market paradigm provides smoother operation, better resource usage and a variety of pricing options.

The required wheeling tariff structure should be able to recover initial investment in wheeling infrastructures, operation and maintenance cost, other wheeling expenses like cost of fuel incurred due to generation rescheduling and re-dispatching, cost incurred in wheeling transactions due to various transmission constraints and cost for future expansion of transmission facilities [3], [4]. On the basis of this tariff structure, wheeling pricing methods can be categorized as embedded cost methods, marginal cost methods and Incremental cost methods [5], [6]. In marginal cost and incremental cost based wheeling pricing, a basic problem is that income is not sufficient for financing the investments (either past or future) [7]. On the other hand, embedded cost methods have been found sufficient to recover the embedded capital investments, operation and maintenance cost (for wheeling) of existing facilities to a particular wheel [8]. Two traditional embedded cost methods used by utility industry are the Postage Stamp method and Contract Path method. These methods are also referred as Rolled-In-Embedded methods in literature [9]. These methods do not reflect actual system operation, as they do not require power flow execution. These methods only represent average system cost; hence do not provide correct economical signals to system users [10]. Other two embedded cost methods, which require execution of power flow, are Boundary flow method and Line-by-Line (MW-mile) method [11]. The convenience obtained with MW-mile method is that, it can be revised to estimate the wheeling prices for actual system usage as close as possible [12].

The best possible use of wheeling resources can be described by monitoring both real power and reactive power, which can be expressed as apparent power flow. In this way the original MW-mile concept can be extended to charge the reactive flow too. This results into an Apparent Flow-mile (AF-mile) method of wheeling pricing, which seems to be more realistic than MW-mile method, as it considers real

flow as well as reactive flow through wheeling facilities [13]-[15]. This paper will focus on the embedded wheeling cost allocation among wheeling facility users (sellers and buyers), employing apparent flow-mile (AF-mile) methods. Three options for AF-mile method have been presented in this paper for analyzing flow based wheeling pricing. These options are used wheeling capacity based pricing, full (used and un-used both) wheeling capacity based pricing and only magnitude and distance based pricing (AF-mile original). Further, the usage based AF-mile methods (used capacity and full capacity based) have been inspected under three categories viz absolute, zero counter flow (ZCF) and reverse.

In the competitive power market the wheeling prices can be allocated to a specific transaction which may be bi-lateral or multi-lateral, without using any power tracing technique. However, in a pool market scenario, the allocation of wheeling prices among various users (sellers and buyers) is not possible without power tracing. Power tracing is a tool which determines the contribution of various users in wheeling a specific transaction over a specific wheeling track.

Bialek tracing is based on the assumption that all nodal inflows are shared proportionally among nodal outflows. This proportional sharing principle followed by Kirchhoff's current law, determines topological distribution factors for both suppliers (sellers) and customers (buyers) to explore their shares in power wheeling [15]-[17]. On the other hand, the generalized distribution factors, traditionally used for power system security and contingency analysis can be adopted for allocating wheeling prices to system users (sellers and buyers) based on an 'use of system approach' [7], [18]. In Kirschen's tracing technique, the buses and network elements are organized in homogeneous groups based on domain, commons, and links [19]-[21]. Besides this, a wheeling transaction can be identified effectively by minimizing the total power-distance in the entire system [22].

The implementation of Bialek tracing for used capacity and full capacity based wheeling pricing methods, has not yet been found in the literature. This paper investigates the effect and feasibility of generalized distribution factors and topological distribution factors based tracing i.e. Rudnick tracing and Bialek tracing respectively, for flow based wheeling pricing. The effect and significance of Rudnick tracing and Bialek tracing on the wheeling prices has been discussed with total seven approaches of wheeling pricing viz AF-mile (in its original form), used absolute AF-mile, used ZCF AF-mile, used reverse AF-mile, full absolute AF-mile, full ZCF AF-mile, and full reverse AF-mile. The proposed approach of wheeling pricing and establishing feasibility and limitations of two tracing techniques has been examined on IEEE-30 bus system [23].

II. PROBLEM FORMULATION

A. Motivation behind Wheeling Prices Allocation through Tracing

Like any other commodity business, in the newly emerged restructured electricity market, wheeling pricing and its allocation among system users, plays a very important role regarding their businesses. Allocation of wheeling prices is important for wheeling facility owner also, because an owner not only has to recover fixed transmission revenue requirements, but also ensures an efficient use of wheeling facilities. To justify the wheeling prices for both, wheeling facility owner and wheeling facility user (seller and/or buyer), flow based pricing methods are mostly adopted by the regulatory bodies/ utilities. Actually, power flow execution based methods of wheeling pricing reflect the actual operating conditions of the wheeling facilities, hence send correct technical and economical signals to the market participants.

Wheeling prices must be distributed over wheeling facility users as well as owners. A major portion of the wheeling prices are bore by wheeling facility users like sellers and buyers, because they use the system for their profits in business and they must have to pay for it. It seems that wheeling facility owner should not be charged for wheeling. But for encouraging wheeling facility users to use the wheeling system efficiently as much as possible, owners should also bear some portion of the wheeling prices that can be seen as user incentive charges. Thus, a proper allocation of wheeling prices among system users and owners must be there. This can be obtained by tracing the flows caused by users for wheeling a particular transaction.

B. Apparent Flow Based Wheeling Pricing Methodologies

A complete recognition has been achieved by apparent flow based pricing methods. It is established that, the effective use of wheeling resources is best measured by monitoring both active and reactive power. Hence, for achieving an effective wheeling pricing, real and reactive both power flows must be considered. In apparent flow based wheeling pricing methods, embedded cost of wheeling facilities is to be allocated to users (sellers and buyers) and owners, considering both active and reactive power loading of transmission network [13]-[15]. This provides more realistic prices for wheeling power. In these methods a wheeling transaction causing more reactive power loading will be charged more than the other transactions. These apparent power flows based wheeling pricing methods have been categorized into three sub methods viz AF-mile original, used capacity based methods and full capacity based methods. In the AF-mile original method, wheeling price allocated to a particular user is computed by considering the apparent power flow caused by that particular user over all the wheeling tracks (lines), embedded cost and length of all wheeling tracks etc. However, in usage based methods, wheeling prices depend upon the actual use of the wheeling facilities.
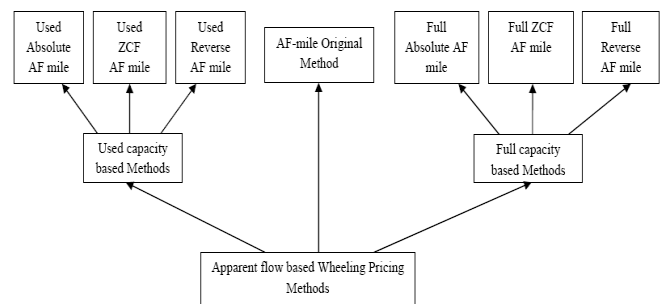

Fig. 1. Apparent flow based wheeling pricing methodologies.

1) AF-mile original

The basic difference between traditional MW-mile original [11] and the present AF-mile original method is that the former one does not consider the reactive power flows over transmission lines caused by various sources. Because of this, it fails to create a realistic view of natural flows, hence pricing based on this method will not send real economical signals to the users as well as owners. Overall wheeling prices will be under estimated in these methods. This problem has been completely overcome in the present AF-mile original method of wheeling pricing.

In AF-mile method, the wheeling prices are considered as the function of magnitude of apparent power flow, the path and the distance travelled by transacted power. This implies that wheeling of same magnitude of apparent power over a wheeling facility of long length will have more weightages towards pricing as compared to short length. However, a wheeling facility may comprise of different types of conductors with different size, thermal rating and cost, thus may lead to different weightages for the same distance travelled by power. This method charges the wheeling prices to sellers and buyers for their apparent flows as follows:

$$WP_s = TTR_S \frac{\sum_{k \in K} c_k L_k AF_{s,k}}{\sum_{s \in S} \sum_{k \in K} c_k L_k AF_{s,k}} \qquad (1)$$

where, WP_s is the wheeling prices computed for s^{th} seller in k\$, TTR_S is the sellers portion of the total transmission revenue, c_k is the cost of kth wheeling facility per MVA per unit length of line, L_k is the length of kth wheeling facility in miles, $AF_{s,k}$ is the apparent flow over k^{th} wheeling facility due to s^{th} seller, K is the set of wheeling facilities and S is the set of sellers.

$$WP_b = TTR_B \frac{\sum_{k \in K} c_k L_k AF_{b,k}}{\sum_{b \in B} \sum_{k \in K} c_k L_k AF_{b,k}} \qquad (2)$$

where, WP_b is the wheeling prices computed for b^{th} buyer in k\$, TTR_B is the buyers portion of the total transmission revenue, c_k is the cost of kth wheeling facility per MVA per unit length of line, L_k is the length of kth wheeling facility in miles, $AF_{b,k}$ is the apparent flow over k^{th} wheeling facility due to b^{th} buyer and B is the set of buyers.

2) Used capacity based wheeling pricing methods

In these methods the wheeling prices are charged to wheeling facility users based on the actual flows they cause. Thus, an encouragement is provided by these methods to users for efficient use of wheeling facilities. These methods have been framed to provide benefit for wheeling facility users. In these methods users have to pay only for the wheeling facility they used for accommodating their transactions. Again, for satisfying owners and motivating users to use wheeling facility efficiently this method comprises of three sub methods viz used absolute, used ZCF and used reverse.

In used absolute AF mile method, users pay for their both the flows i.e. positive and negative. Transmission revenue recovered by this method is more than the fixed transmission cost. Therefore, this method has been easily accepted by wheeling facility owners. However, it does not provide any incentive to the users. Even than a motivation is there for users to use wheeling facilities efficiently and optimizedly so that optimum wheeling prices could be allocated to users. Wheeling prices charged to sellers and buyers under used absolute AF–mile method can be given as:

$$WP(used\ abs)_s = \sum_{k \in K} c_k L_k \frac{|AF_{s,k}|}{AF_{k,\max}} \qquad (3)$$

where, *WP(used abs)$_s$* is the wheeling prices (in k\$) computed for s^{th} seller by used absolute method, c_k is the cost of kth wheeling facility per MVA per unit length of line, modulus of $AF_{s,k}$ is the absolute apparent flow over k^{th} wheeling facility due to s^{th} seller, $AF_{k,max}$ is the line capacity of kth wheeling facility, K is the set of wheeling facilities and S is the set of sellers.

$$WP(used\ abs)_b = \sum_{k \in K} c_k L_k \frac{|AF_{b,k}|}{AF_{k,\max}} \qquad (4)$$

where, *WP(used abs)$_b$* is the wheeling prices (in k\$) computed for b^{th} buyer by used absolute method, modulus of $AF_{b,k}$ is the absolute apparent flow over k^{th} wheeling facility due to b^{th} buyer and B is the set of buyers, $AF_{k,max}$ is the line capacity of kth wheeling facility. Second option for used wheeling capacity based methods is used ZCF method. In this method users have to pay only for their +ve flows (flows which are in the direction of net flow). In this method transmission revenue recovered is always less than the fixed cost, because users are paying only for +ve flows in accommodating their wheeling transactions. Therefor this method is not easily accepted by transmission owner. However, user incentive is definitely there in the form of charges for –ve flows which are not required to be paid by them. Wheeling prices charged to sellers and buyers under used ZCF AF–mile method can be given as:

$$WP(used\ ZCF)_s = \sum_{k \in K} c_k L_k \frac{AF_{s,k}}{AF_{k,\max}} \quad \forall AF_{s,k} > 0 \quad (5)$$

where, *WP(used ZCF)$_s$* is the wheeling prices (in k\$) computed for s^{th} seller by used ZCF method, c_k is the cost of kth wheeling facility per MVA per unit length of line, $AF_{s,k}$ is the apparent flow over k^{th} wheeling facility due to s^{th} seller, K is the set of wheeling facilities and S is the set of sellers, AF_{kmax} is the line capacity of kth wheeling facility.

$$WP(used\ ZCF)_b = \sum_{k \in K} c_k L_k \frac{AF_{b,k}}{AF_{k,\max}} \quad \forall AF_{b,k} > 0 \qquad (6)$$

where, *WP(used ZCF)$_b$* is the wheeling prices (in k\$) computed for b^{th} buyer by used ZCF method, $AF_{b,k}$ is the apparent flow over k^{th} wheeling facility due to b^{th} buyer and B is the set of buyers, AF_{kmax} is the line capacity of kth wheeling facility. Third option for used wheeling capacity based methods is used reverse method. This method is most suited for users as in this method users have to pay only for their positive flows and also get credit for their –ve flows as a reward for helping in releasing congestion. Therefor wheeling prices paid by users are even less than those in used

ZCF method. However there is no advantage for wheeling facility owners to opt this method. Wheeling prices charged to sellers and buyers under used reverse AF–mile method can be given as:

$$WP(used\ rev)_s = \sum_{k \in K} c_k L_k \frac{AF_{s,k}}{AF_{k,\max}} \qquad (7)$$

where, $WP(used\ rev)_s$ is the wheeling prices (in k$) computed for s^{th} seller by used reverse method, $AF_{s,k}$ is the apparent flow over k^{th} wheeling facility due to s^{th} seller, K is the set of wheeling facilities and S is the set of sellers, $AF_{k,max}$ is the line capacity of kth wheeling facility.

$$WP(used\ rev)_b = \sum_{k \in K} c_k L_k \frac{AF_{b,k}}{AF_{k,\max}} \qquad (8)$$

where, $WP(used\ rev)_b$ is the wheeling prices (in k$) computed for b^{th} buyer by used reverse method, $AF_{b,k}$ is the apparent flow over k^{th} wheeling facility due to b^{th} buyer and B is the set of buyers, $AF_{k,max}$ is the line capacity of kth wheeling facility.

3) Full capacity based wheeling pricing methods

In these methods the wheeling prices are charged to wheeling facility users, based on whole wheeling capacity. This means that users are required to pay not only for the actual line flows they cause, but also for the wheeling capacity which is not being used by them. In this way these methods guarantee the full recovery of total transmission revenue incurred as the embedded cost. However, these methods do not motivate an efficient use of wheeling facilities. These methods are good from wheeling facility owner's point of view, as these ensure fixed cost recovery, where as the snag with these methods is that they do not provide any charm for the users. These pricing methods have been described for three conditions of wheeling flows. First is for pricing the flows in both the directions (+ve and –ve flows), second is pricing for only +ve flows and third one is pricing for positive flows and getting credit for –ve flows. On the basis of these three conditions this method can be categorized into three sub methods viz full absolute, full ZCF and full reverse AF-mile method.

In the full capacity based absolute method, users pay not only for positive flows (the flows which are in the direction of net flow) but also pay for negative flows (all the flows which are in the opposite direction to the net flow) they cause. Wheeling prices computed for sellers and buyers are as follows:

$$WP(full\ abs)_s = \sum_{k \in K} c_k L_k \frac{|AF_{s,k}|}{\sum_{s \in S} |AF_{s,k}|} \qquad (9)$$

where, $WP(full\ abs)_s$ is the wheeling prices (in k$) computed for s^{th} seller by full capacity based absolute method, C_k is the cost of kth wheeling facility per MVA per unit length of line, modulas of $AF_{s,k}$ is the absolute apparent flow over k^{th} wheeling facility due to s^{th} seller, K is the set of wheeling facilities and S is the set of sellers.

$$WP(full\ abs)_b = \sum_{k \in K} c_k L_k \frac{|AF_{b,k}|}{\sum_{b \in B} |AF_{b,k}|} \qquad (10)$$

where, $WP(full\ abs)_b$ is the wheeling prices (in k$) computed for b^{th} buyer by full capacity based absolute method, modulas

of $AF_{b,k}$ is the absolute apparent flow over k^{th} wheeling facility due to b^{th} buyer and B is the set of buyers. Again, second option for full capacity based wheeling pricing methods is ZCF method. In this method users have to pay only for their +ve flows that mean they don't have to pay for –ve flows. Wheeling prices computed for sellers and buyers are as follows:

$$WP(full\ ZCF)_s = \sum_{k \in K} c_k L_k \frac{AF_{s,k}}{\sum_{s \in S} AF_{s,k}} \qquad \forall AF_{s,k} > 0 \qquad (11)$$

where, $WP(full\ ZCF)_s$ is the wheeling prices (in k$) computed for s^{th} seller by full ZCF method, $AF_{s,k}$ is the apparent flow over k^{th} wheeling facility due to s^{th} seller, K is the set of wheeling facilities and S is the set of sellers.

$$WP(full\ ZCF)_b = \sum_{k \in K} c_k L_k \frac{AF_{b,k}}{\sum_{b \in B} AF_{b,k}} \qquad \forall AF_{b,k} > 0 \qquad (12)$$

where, $WP(full\ ZCF)_b$ is the wheeling prices computed for b^{th} buyer by full ZCF method, $AF_{b,k}$ is the apparent flow over k^{th} wheeling facility due to b^{th} buyer and B is the set of buyers. Third option for full capacity based wheeling pricing methods is full reverse method. In this method users only have to pay for their +ve flows and also get credit for their –ve flows. Wheeling prices computed for sellers and buyers are as follows:

$$WP(full\ rev)_s = \sum_{k \in K} c_k L_k \frac{AF_{s,k}}{\sum_{s \in S} AF_{s,k}} \qquad (13)$$

where, $WP(full\ rev)_s$ is the wheeling prices (k$) computed for s^{th} seller by full reverse method, $AF_{s,k}$ is the apparent flow over k^{th} wheeling facility due to s^{th} seller, K is the set of wheeling facilities and S is the set of sellers.

$$WP(full\ rev)_b = \sum_{k \in K} c_k L_k \frac{AF_{b,k}}{\sum_{b \in B} AF_{b,k}} \qquad (14)$$

where, $WP(full\ rev)_b$ is the wheeling prices (in k$) computed for b^{th} buyer by full reverse method, $AF_{b,k}$ is the apparent flow over k^{th} wheeling facility due to b^{th} buyer and B is the set of buyers.

III. RESEARCH METHODOLOGY

The research methodology adopted for the work carried out in the present paper has been shown in Fig. 2. Pool based power market data is generated in the required form from the data given. AC load flow program is executed. Two tracing techniques i.e. Rudnick's tracing and Bialek's tracing, have been implemented to determine share or contribution of each supplier/seller and buyer in every wheeling facility flow. Topological distribution factors have been computed through Bialek's tracing, while generalized distribution factors for sellers and buyers have been determined through Rudnick's tracing. Then for comparing and establishing significance of various wheeling pricing methods from seller's and buyer's point of view, seven individual wheeling pricing methods were employed. Through these methods finally wheeling prices have been allocated to each seller and buyer in the pool power market.
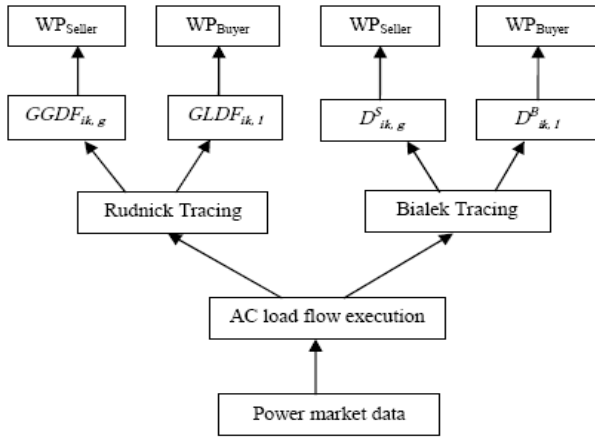
Fig. 2. Research methodology for the proposed work.

A. Rudnick's Tracing

To achieve a given economics of scale of power wheeling networks, not only an adequate revenue reconciliation method is required but also an appropriate allocation of wheeling prices amongst all users (including the user actually owing the power wheeling network). Proper allocation of wheeling prices requires power tracing. Rudnick presented a very practical approach of power tracing through three distribution factors which reasonably ensures system security with contingency analysis. Rudnick tracing is based on 'a use of system' approach. On the basis of this tracing generalized distribution factors are framed to determine maximum transaction related power flows [7], [24]. In rudnick tracing, besides the traditional distribution factors, generalized generation and generalized load distribution factors (GGDF, GLDF) have been implemented, representing three different relations: generation shift distribution factors (GSDF) representing the incremental change which is produced in any line by changing the power injection in any bus (except in a reference bus); GGDF factors representing the impact of change in generation at any generation bus on to the power flows over any line; GLDF factors representing the impact of change in load at any load bus on to the power flow over any line. With the help of these three factors Rudnick tracing provides an effective approach to allocate wheeling prices amongst system users.

GSDF has been determined through a sensitivity analysis and indicates a relation between a change in power injection (generator minus load) in a given bus and a change in power flow in a particular line. Numbers of GSDFs depend upon the number of lines to be monitored. GSDFs are used to calculate line flows after shift of generation, which is quite simpler than running a complete load flow for that shift. GSDF can be defined by the following equations:

$$\Delta F_{i-k} = GSDF_{i-k} \Delta G_g \qquad (15)$$

$$\text{such that, } \Delta G_g + \Delta G_R = 0 \qquad (16)$$

Equation (16) shows that total generation of a system remains unchanged. Where, ΔG_g is change in generation at g^{th} generation bus excluding the reference bus R. ΔG_R is change in generation in reference bus R. ΔF_{i-k} is incremental change in flow over line $i-k$ (from bus i to bus k) due to shifting of generation. $GSDF_{ik}$ is a proportional constant for line $i-k$, due to shift of generation on g^{th} generator bus. From (15) it can be

written as,

$$GSDF_{i-k,g} = \frac{\Delta F_{i-k}}{\Delta G_g} = \frac{\Delta I_{i-k}}{\Delta I_g} \qquad (17)$$

where ΔI_{i-k} is the change in current in line $i-k$ due to shift of generation ΔG_g from reference bus R to g^{th} bus, ΔI_g is the change in injection current into g^{th} bus. Voltages at all the buses have been taken as 1pu. Through the definition of reactance matrix X,

$$\Delta g_{i-k} = \frac{\Delta V_i - \Delta V_k}{x_{i-k}} \qquad (18)$$

$$\Delta V_i = \Delta I_g x_{i-g} = \frac{\Delta I_g x_{i-g} - \Delta I_g x_{k-g}}{x_{i-k}} \qquad (19)$$

$$\Delta I_{i-k} = \Delta I_g \frac{x_{i-g} - x_{k-g}}{x_{i-k}} \qquad (20)$$

where x_{i-g} and x_{k-g} are the elements of reactance matrix and x_{i-k} is line reactance. Substituting (20) into (17) we get,

$$GSDF_{i-k,g} = \frac{x_{i-g} - x_{k-g}}{x_{i-k}} \qquad (21)$$

Using *GSDFs*, only the change in flow on a particular line due to generation shift can be calculated. However, for wheeling pricing actual flows instead of their incremental values are required. For this purpose, set of *GGDFs* have been defined by the following equation:

$$F_{i-k} = \sum_g GGDF_{i-k,g} G_g \qquad (22)$$

where the summation goes through all the generator buses, and F_{i-k} is actual apparent power flow over line $i-k$, G_g is generation on g^{th} bus and $GGDF_{i-k,g}$ is generalized generation distribution factor for line $i-k$ due to g^{th} generator.

GGDF represents the portion of generation supplied by g^{th} generator which flows over a particular line. This concept has been used in the present paper to determine the portion of power supplied by a particular seller which flows over a particular line. So that, total wheeling pricing for a particular seller can be determined for its total power being wheeled over different lines. The generalized generation distribution factor for line $i-k$ due to g^{th} generator can be computed as follows [18]:

$$GGDF_{ik,g} = GSDF_{ik,g} + GGDF_{ik,R} \qquad (23)$$

$$GGDF_{ik,R} = \frac{F_{ik} - \sum_{g,g \neq R}^{N} GSDF_{ik,g} G_g}{\sum_{g \in N} G_g} \qquad (24)$$

where, F_{ik} is actual apparent power flow over line $i-k$, G_g is generation on g^{th} bus, R is reference bus and N is total no. of generator buses. These factors are formulated overruling the need of reference bus and limitation of constant total generation.

According to the notion of *GGDF*, generalized load distribution factors have also been formulated to provide the portion of load which flows over a particular line. *GLDF* shows the amount of load supplied by l^{th} load bus, which

flows over a particular line *i-k* and can be presented as follows:

$$GLDF_{ik,l} = GLDF_{ik,R} - GSDF_{ik,l} \qquad (25)$$

$$GLDF_{ik,R} = \frac{F_{ik} - \sum\limits_{l,l \neq R}^{M} GSDF_{ik,l} \, L_l}{\sum\limits_{l \in M} L_l} \qquad (26)$$

where, L_l is load on l^{th} load bus, M is total number of load buses.

B. Bialek's Tracing

In Bialek's tracing, allocation of wheeling facility usage (transmission line flows) to individual users is carried out by analyzing topology of line flows. This method tracks the path where output of every generator goes and the path through which the input to every load comes from. Bialek tracing is based on proportional sharing principle. According to this principle, each outgoing flow through every node is in proportion of every incoming flow at that node. The resulting topological distribution factor determines the wheeling facility usage by any seller and/or buyer by adding the contribution of each seller or buyer to every line flow [16]. Bialek's tracing proposes two algorithms on the basis of which employing proportional sharing principle, the flows due to every seller and buyer have been traced for allocating wheeling prices.

Upstream algorithm is named for the procedure which is adopted to determine portion of power generation at any generator bus (seller node) which flows in a particular line by considering all the inflows at that node. The total apparent power flow S_i through node *i* may be expressed as follows looking all the inflows:

$$S_i = \sum_{j \in u(i)} |S_{ij}| + S_{Gi} \qquad for\ i = 1,2,3,....n \qquad (27)$$

where *u (i)* is the set of nodes supplying directly node *i*, S_{ij} is the apparent power flow towards node *i* in line *j-i* and S_{Gi} is the generation at node *i* and *n* is total number of buses.

Since the electricity is indistinguishable and each of the outflows down the line from node *i* is dependent only on the voltage gradient and impedance of line. So according to the proportional sharing principle each flow leading the node *i* contains the same proportion of the inflows as the total nodal flow S_i. Here we are considering all the inflows, so the line flows $|S_{ij}| = |S_{ji}|$ can be related to the nodal flow at node *j* by putting $|S_{ij}| = x_{ji} S_j$ where $x_{ji} = |S_{ji}|/S_j$. Thus,

$$S_i = S_{Gi} + \sum_{j \in u(i)} x_{ji} S_j \qquad (28)$$

$$or \ S_i - \sum_{j \in u(i)} x_{ji} S_j = S_{Gi} \qquad (29)$$

$$or \ A_u S = S_G \qquad (30)$$

where A_u is the upstream distribution matrix (*n* x *n*), *S* is vector of nodal through flows and S_G is vector of generations. Here ij^{th} element of A_u will be,

$$[A_u]_{ij} = \begin{cases} 1 & for\ i = j \\ -x_{ji} & for\ j \in u(i) \\ 0 & otherwise \end{cases} \qquad (31)$$

Here it must be noted that obviously elements of A_u matrix will depend upon topological configuration of the system and A_u will be a non-symmetric sparse matrix. For some topological configurations inverse of this A_u matrix does not exist. If A_u^{-1} exists, then $S = A_u^{-1} S_G$ and its ith element will be

$$S_i = \sum_{g=1}^{n} [A_u^{-1}]_{ig} S_{Gg} \qquad for\ i = 1,2,........n \qquad (32)$$

eq. (32) shows that contribution of gth system generator to i^{th} nodal flow is equal to $[A_u^{-1}]_{ig} S_{Gg}$. Now it must be noted that the same nodal flow S_i is equal to the sum of load demand S_{Li} and all outflows leaving node *i*. Therefore a line flow in line *i-k* from node *i* can be found, by using the proportional sharing principle as,

$$|S_{ik}| = \frac{|S_{ik}|}{S_i} S_i \qquad (33)$$

$$|S_{ik}| = \frac{|S_{ik}|}{S_i} \sum_{g=1}^{n} [A_u^{-1}]_{ig} S_{Gg} \qquad (34)$$

$$or \ |S_{ik}| = \sum_{g=1}^{n} D_{ik,g}^{G} S_{Gg} \qquad for\ all\ k \in d(i) \qquad (35)$$

where *d (i)* is the set of nodes supplied directly from node *i*, and $D_{ik,g}^{G}$ is topological generation distribution factor which can be used to provide portion of generation due to g^{th} generator which flows in line *ik*. The nodal through flow S_i can be written as the sum of outflows,

$$S_i = \sum_{l \in d(i)} |S_{il}| + S_{Li} \qquad for\ i = 1,2,3.....n \qquad (36)$$

$$S_i = S_{Li} + \sum_{l \in d(i)} x_{li} S_l \qquad for\ i = 1,2........n \qquad (37)$$

where, *d (i)* is the set of all the nodes supplied directly from node *i* and $x_{li} = |S_{li}|/S_l$.

This can be written as

$$S_i - \sum_{l \in d(i)} x_{li} S_l = S_L \qquad (38)$$

$$or \ A_d S = S_L \qquad (39)$$

where, A_d is the downstream distribution matrix (*n* x *n*), *S* is vector of nodal through flows and S_L is vector of loads. Here il^{th} element of A_d will be,

$$[A_d]_{il} = \begin{cases} 1 & for\ i = l \\ -x_{li} = -|S_{li}|/S_l & for\ l \in d(i) \\ 0 & otherwise \end{cases} \qquad (40)$$

It must be noted here that A_d is also a sparse and non-symmetric matrix. These two non-symmetric matrices when added, gives the symmetric nodal admittance matrix. If inverse of A_d exists then $S = A_d^{-1} S_L$ and its ith element is equal to

$$S_i = \sum_{k=1}^{n} \left[A_d^{-1} \right]_{ik} S_{Lk} \qquad i = 1, 2, \ldots, n \qquad (41)$$

Eq (15) shows how the nodal power S_i distributed between all the loads in the system. On the other hand, the same S_i is equal to the sum of the generation at node i and all the inflows in lines entering this node. Therefore, the inflow to node i from line i-j can be calculated using proportional sharing principle as

$$\left| S_{ij} \right| = \frac{\left| S_{ij} \right|}{S_i} S_i = \frac{\left| S_{ij} \right|}{S_i} \sum_{k=1}^{n} \left[A_d^{-1} \right]_{ik} S_{Lk} \qquad (42)$$

$$= \sum_{k=1}^{n} D_{ij,k}^{L} S_{Lk} \qquad for\ all\ j \in u(i) \qquad (43)$$

where $D_{ij,k}^{L}$ is the topological load distribution factor which represents the portion of kth load that flows in line i-j. In this way Bialek tracing provides contribution of a seller (generator) or buyer (load) in a flow over a particular line.

IV. RESULTS AND DISCUSSION

The embedded cost based methods for wheeling pricing are found sensitive for the distance and direction of power flows. If users are paying wheeling prices for their power flows in the same direction as that of net flow these are called positive flow prices. However, some flows are there in the system which are in the opposite direction to that of net flow, these are called negative or counter flows. The wheeling prices for such flows are called counter flow prices. In power market (pool, bilateral and multilateral) counter flow prices are always debatable. However, in this paper, to discuss this issue three different wheeling pricing methodologies have been used which are absolute pricing, zero counter flow pricing and reverse pricing. These three pricing schemes have been implemented with full wheeling capacity and used wheeling capacity based approaches.

In order to determine wheeling prices for each seller and buyer in a pool power market paradigm, power tracing has been proved to be a very useful tool. However, these tracing methods have their limitations and sometimes are not applicable to determine the required power wheeled by a seller or a buyer. In the present paper, two tracing methods have been implemented to examine their feasibility for seven wheeling pricing methods, under pool power market environment. Two tracing methods named as Rudnick tracing and bialek tracing have applied for determining apparent power wheeled through sellers and buyers. Then AF-mile original method used wheeling capacity methods and full wheeling capacity methods, thus total seven wheeling pricing methods have been implemented to allocate wheeling prices among each seller and buyer. In this work, it has been verified that full capacity based methods are only for recovering fixed (pre-estimated) wheeling cost while, used capacity based methods determine the wheeling prices only for the power wheeled by any seller or buyer. Thus, used capacity based wheeling pricing methods are found more economical for users (specifically for sellers) then full capacity based methods. Since buyers receive power from various nodes hence power wheeled by a buyer is always more than power wheeled by a seller. Thus, used capacity

based wheeling pricing is not found economical for buyers as compared to full capacity based wheeling pricing.

The present approach for establishing applicability of two most profound tracing methods and then implementation of seven wheeling pricing methods have been examined on IEEE-30 bus system based pool power market. As per the given data, the total annualized cost of wheeling structure for IEEE-30 bus system is 8851.84 k$. This is the total fixed wheeling cost for the present pool power market, which is to be recovered by sellers and buyers. In the present power market, total numbers of sellers of electric power are 06 and total numbers of buyers are 21. Total numbers of wheeling tracks are 41, through which power was wheeled in the market. In the present work, seller's contribution has been taken as 30% of total wheeling cost while buyer's share has been considered as 70% of total wheeling cost. This consideration has been found quite justifiable, as amount of power wheeled by buyers is more than that wheeled by sellers. In this way the wheeling services owner must recover 2655.55 k$ from sellers and 6196.29 k$ from buyers for providing wheeling infrastructure for their power transactions.

A. Rudnick Tracing

The basic concept of Rudnick tracing is, to determine the power flow contribution of each seller or buyer over each wheeling track (line) on the basis of some distribution factors. Through these generalized distribution factors wheeling prices to be paid by sellers and buyers have been computed. Table I and Table II show the wheeling prices paid by sellers and buyers respectively in IEEE-30 bus system based pool power market through Rudnick tracing. Table I shows the wheeling prices for all the 06 sellers determined under 07 wheeling pricing methods. The main advantage of Rudnick tracing is that, it is capable of finding power flows in both the directions, means in the direction of net flow as well as in the direction of opposite flow. All the positive values of wheeling prices in Table I and Table II refer the prices paid by sellers and buyers respectively for various wheeling pricing methods. It can be seen in Table I, that wheeling price for seller S5 is -2.83 k$ under the used reverse AF-mile approach of wheeling pricing. Here 'minus' sign indicates that in this particular approach of wheeling pricing, the seller S5 will not pay any amount, instead will get 2.83 k$ from buyers. Similarly, it can be seen in Table I that seller S8 and S11 will get (instead of paying), 11.57 k$ and 33.68 k$ respectively under full capacity based reverse AF-mile method of wheeling pricing. It has been also verified from Table I that AF-mile original method, full absolute, full ZCF and full reverse methods of wheeling pricing recover only fixed wheeling prices from sellers which is 2655.55 k$ for the present case.

Sellers have to pay lesser price for their wheeling power as compared to fixed cost of wheeling in used absolute AF-mile approach. It has been depicted in Table I that wheeling prices to be paid by sellers are 2381.67 k$ which is lesser than fixed cost of 2655.55 k$. However, it is not true in the case of buyers. Table II shows that under used absolute AF-mile approach calculated wheeling prices which are to be paid by buyers are 6853.03 k$ which is higher than the fixed cost of wheeling i.e. 6196.29 k$.

The used reverse AF- mile approach of wheeling pricing has been found to be best amongst all these seven methods of wheeling pricing from users point of view.

TABLE I: ALLOCATION OF WHEELING PRICES ACROSS SELLERS USING RUDNICK TRACING

S.No.	Sellers	Methodologies for Wheeling Pricing (in k$)						
		AF-mile original	used abs	used ZCF	used reverse	full abs	full ZCF	full reverse
1	S1	1775.69	1499.68	1498.53	1497.38	1633.18	1868.71	2183.69
2	S2	389.89	313.58	290.98	268.38	368.68	392.9	425.29
3	S5	123.79	124.11	60.64	-2.83	159.65	128.54	86.92
4	S8	138.82	166.41	83.32	0.23	188.45	102.87	-11.57
5	S11	99.29	108.71	65.63	22.55	146.35	69.33	-33.68
6	S13	128.06	169.18	110.15	51.12	159.24	93.2	4.9
	Sum	**2655.55**	**2381.67**	**2109.26**	**1836.83**	**2655.55**	**2655.55**	**2655.55**

TABLE II: ALLOCATION OF WHEELING PRICES ACROSS SELLERS USING RUDNICK TRACING

S.No.	Buyers	Methodologies for Wheeling Pricing (in k$)						
		AF-mile original	used abs	used ZCF	used reverse	full abs	full ZCF	full reverse
1	B2	165.22	178	92.35	6.69	174	130.97	63.31
2	B3	47.97	37.66	14.18	-9.3	33.84	20.61	-0.18
3	B4	123.07	105.54	63.31	21.08	100.41	85.99	63.32
4	B5	1555.6	1553.86	1385.35	1216.84	1362.93	1326.61	1269.52
5	B7	396.01	426.11	335.27	244.43	394.85	453.07	544.61
6	B8	541.11	630.83	503.99	377.15	585.07	634.32	711.73
7	B10	177.51	177.66	140.25	102.84	160.45	159.14	157.07
8	B12	319.17	325.26	213.53	101.78	293.64	239.51	154.43
9	B14	191.13	237.78	154.89	71.98	214.09	170.46	101.88
10	B15	248.77	297.5	222.07	146.64	268.05	240.21	196.44
11	B16	105.26	109.62	83.86	58.1	98.76	94.12	86.82
12	B17	287.78	298.52	256.48	214.44	269.98	288.86	318.55
13	B18	104.44	124.3	101.6	78.9	111.71	102.8	88.79
14	B19	318.98	373.37	336.31	299.25	331.71	328.11	322.45
15	B20	72.5	82	67.97	53.94	73.89	80.93	92
16	B21	600.33	671.04	571.27	471.5	606.67	589.74	563.13
17	B23	100.59	130.23	103.89	77.55	117.04	114.02	109.25
18	B24	279.63	348.42	306.32	264.2	316.41	343.75	386.73
19	B26	128.51	167.54	141.46	115.38	152.14	167.7	192.18
20	B29	79.65	105.33	81.61	57.89	96.44	109.28	129.47
21	B30	353.05	474.46	391.19	307.92	434.2	516.08	644.79
	Sum	**6196.29**	**6853.03**	**5567.16**	**4279.2**	**6196.29**	**6196.29**	**6196.29**

The least wheeling prices have been computed for sellers i.e. 1836.83 k$ under used reverse AF-mile method which is far less as compared to fixed cost of 2655.55 k$. Thus, this method is not good from wheeling services owner point of view and will not be easily accepted by them. For buyers also this method is best. The least wheeling prices i.e. 4279.20 k$ were found to be paid by buyers under this method which is quite lesser than fixed wheeling cost for buyers i.e. 6196.29 k$. The percentage of total wheeling cost which is being paid by each seller and buyer has been illustrated graphically in Fig. 1 and Fig. 2 respectively. In both these figures wheeling prices paid by sellers and buyers were determined using above discussed 07 wheeling pricing methods. The negative value of wheeling price for seller S11 under full reverse AF-mile method can be seen in Fig. 1.

B. Bialek Tracing

In the pool power market environment, at each node, a seller, or a buyer or both can be existed. Hence each outflow or inflow at a node is due to either a seller or a buyer or both. The basic principle of bialek tracing is that, every outflow at any node will be in proportion to all the inflows at that node.

These outflows and inflows are due to sellers and buyers. However, it must be noted here that, in bialek tracing, the contribution of any seller or any buyer to any nodal flow depends upon inverse of upstream distribution matrix i.e. A_u^{-1} and inverse downstream distribution matrix i.e. A_d^{-1} respectively. Mathematically it has been found that, all the elements of either A_u^{-1} or A_d^{-1} are either zero or positive. This means that contribution of any seller or buyer in a negative flow cannot be determined by bialek tracing. Thus, all the used capacity based methods whether it is used absolute or used ZCF or used reverse, will provide same wheeling prices for sellers and buyers in bialek tracing. Table III shows that wheeling prices of 2114.73 k$ have been computed by all the three used capacity based methods.

This can also be observed in Table III, that wheeling prices computed for seller S5 with all the seven wheeling pricing methods are zero. This is because, all the elements of A_u^{-1} matrix due to seller S5 for all the lines are mathematically found to be zero. This shows the feasibility of bialek tracing towards a particular topological power network. Bialek tracing has also been examined on some other power wheeling networks e.g. Indian utilty 62-bus system, but

results were found not defined. This means some elements of A_u^{-1} matrix and A_d^{-1} matrix were found not defined. That is why the present work has been carried out on IEEE-30 bus system for which the results of bialek tracing were existed.
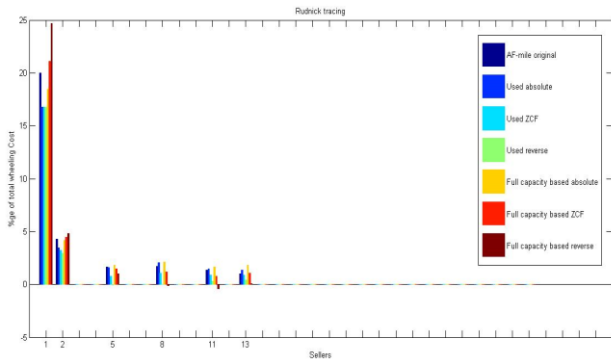

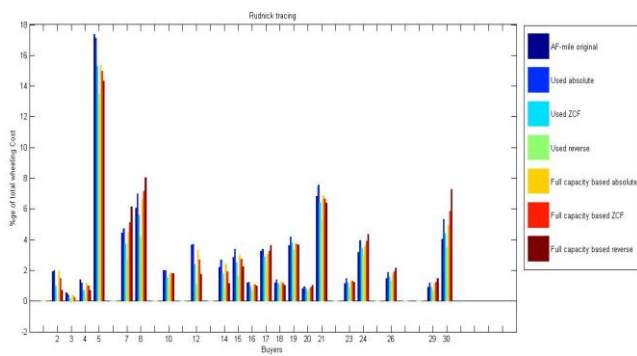Fig. 3. Wheeling prices allocated to sellers on IEEE-30 bus system using Rudnick tracing.

Fig. 4. Wheeling prices allocated to buyers on IEEE-30 bus system using Rudnick tracing.
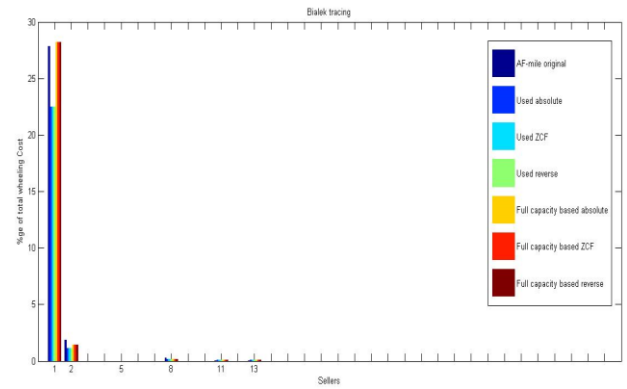

Fig. 5. Allocation of wheeling prices for sellers on IEEE-30 bus system using Bialek tracing.
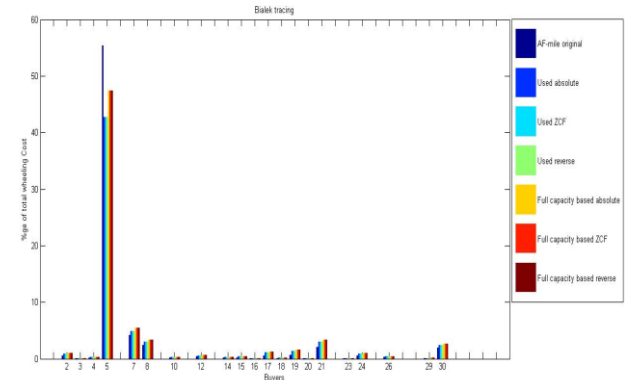

Fig. 6. Allocation of wheeling prices for buyers on IEEE-30 bus system using Bialek tracing.

TABLE III: WHEELING PRICES ALLOCATED TO SELLERS IN IEEE-30 BUS SYSTEM BASED POOL POWER MARKET USING BIALEK TRACING

S.No.	Sellers	Methodologies for Wheeling Pricing (in k$)						
		AF-mile original	Used abs	Used ZCF	Used reverse	full abs	full ZCF	full reverse
1	S1	2463	1990	1990	1990	2488.9	2488.9	2488.9
2	S2	164.53	99.32	99.32	99.32	134.74	134.74	134.74
3	S5	0	0	0	0	0	0	0
4	S8	24	11.04	11.04	11.04	13.87	13.87	13.87
5	S11	1.51	5.8	5.8	5.8	7.29	7.29	7.29
6	S13	2.51	8.57	8.57	8.57	10.76	10.76	10.76
	Sum	2655.55	2114.73	2114.73	2114.73	2655.55	2655.55	2655.55

TABLE IV: WHEELING PRICES ALLOCATED TO BUYERS IN IEEE-30 BUS SYSTEM BASED POOL POWER MARKET USING BIALEK TRACING

S.No.	Buyers	Methodologies for Wheeling Pricing (in k$)						
		AF-mile original	Used abs	Used ZCF	Used reverse	full abs	full ZCF	full reverse
1	B2	41.62	78.65	78.65	78.65	87.38	87.38	87.38
2	B3	1.04	1.41	1.41	1.41	1.34	1.34	1.34
3	B4	17.9	24.54	24.54	24.54	27.47	27.47	27.47
4	B5	4910.24	3776.48	3776.48	3776.48	4195.81	4195.81	4195.81
5	B7	368.14	429.86	429.86	429.86	477.59	477.59	477.59
6	B8	193.36	267.05	267.05	267.05	296.7	296.7	296.7
7	B10	18.79	24.09	24.09	24.09	26.77	26.77	26.77
8	B12	33.11	50.91	50.91	50.91	56.56	56.56	56.56
9	B14	11.55	23.01	23.01	23.01	25.57	25.57	25.57
10	B15	21.59	35.95	35.95	35.95	39.94	39.94	39.94
11	B16	4.58	7.48	7.48	7.48	8.31	8.31	8.31
12	B17	51	100.81	100.81	100.81	112	112	112
13	B18	6.3	12.56	12.56	12.56	13.95	13.95	13.95
14	B19	57.64	122.64	122.64	122.64	136.26	136.26	136.26
15	B20	2.78	4.58	4.58	4.58	5.09	5.09	5.09
16	B21	187.17	267.32	267.32	267.32	297	297	297
17	B23	5.27	9.22	9.22	9.22	10.25	10.25	10.25
18	B24	53.47	81.17	81.17	81.17	90.18	90.18	90.18
19	B26	26.06	36.27	36.27	36.27	40.3	40.3	40.3
20	B29	7.94	9.41	9.41	9.41	10.45	10.45	10.45
21	B30	176.73	213.64	213.64	213.64	237.36	237.36	237.36
	Sum	6196.29	5577.05	5577.05	5577.05	6196.29	6196.29	6196.29

Bialek tracing also provides wheeling prices same as that of fixed wheeling cost i.e. 2655.55 k$ and 6196.29 k$ for sellers and buyers respectively under AF-mile original, full absolute, full ZCF and full reverse methods of wheeling pricing, which can be seen in Table III and Table IV. A very important observation made from the results is that wheeling prices for sellers under used absolute method of wheeling

pricing were higher in Rudnick tracing than that of bialek tracing. This is because, in bialek tracing approach, sellers have to pay only for their +ve flows. On the other hand, with used reverse method of wheeling pricing, bialek tracing approach provides higher wheeling prices as compared to Rudnick tracing, as in bialek tracing credit for counter flows cannot be seeked by users.

TABLE V: COMPARISON OF BOTH TRACING TECHNIQUES WITH ALL PRICING METHODS

Tracing	Users	Methodologies for Wheeling Pricing (in k$)						
		AF-mile original	Used abs	Used ZCF	Used reverse	full abs	full ZCF	full reverse
Bialek Tracing	Seller	2655.55	2114.75	2114.75	2114.75	2655.55	2655.55	2655.55
	Buyer	6196.29	5577.03	5577.03	5577.03	6196.29	6196.29	6196.29
	Total	**8851.84**	**7691.78**	**7691.78**	**7691.78**	**8851.84**	**8851.84**	**8851.84**
Rudnick Tracing	Seller	2655.55	2381.67	2109.26	1836.83	2655.55	2655.55	2655.55
	Buyer	**6196.29**	6853.03	5567.16	4279.2	6196.29	6196.29	6196.29
	Total	**8851.84**	**9234.7**	**7676.42**	**6116.03**	**8851.84**	**8851.84**	**8851.84**

Table V shows a comparative study of bialek tracing and Rudnick tracing for all seven methods of wheeling pricing. Total wheeling prices for both sellers and buyers with used reverse wheeling pricing method, are determined as 7691.78 k$ in bialek tracing and 6116.03 k$ in Rudnick tracing. In Rudnick tracing flows are considered in both the directions hence due to the credit of counter flows, the total wheeling prices with used reverse wheeling pricing method in Rudnick tracing have been found lesser than that which was found in bialek tracing. Because in bialek tracing no credit can be seeked by users due to counter flows. Similarly wheeling prices obtained with used ZCF method are found to be same in bialek tracing as well as in Rudnick tracing, because prices for only +ve flows are to be considered in used ZCF method.

It can also be seen in Table V that in full absolute, full ZCF and full reverse methods the wheeling prices computed for either sellers or buyers have been found fixed for all three methods. Because wheeling prices have been charged for full capacity (used and unused both) and mathematically summation of prices for all the sellers and all the buyers become fixed. This is so because a flow which is +ve for one seller may be –ve for another seller. Thus, summation of prices for all the sellers would have become same in three different approaches viz full absolute, full ZCF and full reverse.

V. CONCLUSION

Embedded cost based wheeling pricing methods have been found sufficient enough in meeting total transmission revenue requirements. Further usage based embedded methods are found capable to send correct economic signals to power market owner and users as they are based on actual operation of power market. In order to consider the significance of real and reactive power both apparent power flow based wheeling pricing techniques have been presented in the paper. To carry out the analysis on a big canvas seven wheeling pricing techniques (AF-mile based) have been presented in the paper. Out of these methods, used absolute wheeling pricing method has been found most pertinent for wheeling facility provider (owner). However, used reverse method was found most reasonable for wheeling facility users.

The present paper establishes the applicability of power tracing in allocation of wheeling prices amongst users in a pool power market scenario. For this purpose, two tracing techniques have been implemented viz Rudnick tracing and Bialek tracing.

The bialek tracing has not been applied with used capacity based wheeling pricing methods yet. This is first time, that applicability of bialek tracing has been judged with both used capacity based wheeling pricing methods and full capacity based wheeling pricing methods. Since bialek tracing responds to power flows (both inflows and outflows at any node) in one direction only, it provides same results for used ZCF and used reverse approach of wheeling prices. As a further research the implementation of tracing to determine wheeling prices for bilateral and multilateral transactions has been proposed.

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