Proposal of a Network-Based Minority Game Model with Observation and Modification Processes

Keita Nishimoto, Ivan Tanev, and Katsunori Shimohara

***Abstract*—We propose a new model, the *network-based Minority Game with observation and modification processes (NMG)*, to investigate and analyze the effect of local information exchange between individuals on whole system dynamics in multi-agent simulations. The Minority Game is an *N*-player game which captures the collective behavior of adaptive agents in an idealized situation where they compete for some finite resource. The NMG extends the Minority Game by introducing a social network between agents and adding observation and modification processes into the agent's decision making procedure. Performing multi-agent simulations of the NMG, we discovered that increasing the out-degree of the network, that is, the number of agents from which an agent can acquire information, has a negative effect on agents' wealth and can also affect wealth inequality between agents. Moreover, our results showed that social efficiency in the NMG differs remarkably from the original Minority Game.**

***Index Terms*—Multi-agent simulation, minority game.**

I. Introduction

The Minority Game (MG) - proposed by Challet and Zhang under the inspiration of the El Farol bar problem [1] - is an N-player game that represents the collective behavior of adaptive agents in an idealized situation where they have to compete for some finite resource. Each of the N agents independently chooses one of two alternatives, and those agents which picked the minority choice win and are awarded a point [2]. The Minority Game has been studied actively in various fields such as Multi-Agent Systems [3] and congestion control [4], because of its simplicity and emergent characteristics (e.g., the emergence of cooperation among agents and phase transitions with symmetry breaking [5]).

In the original model proposed by Challet and Zhang, agents make their choice on basis of a global past record, without local information exchange. However, in the real world, people in competitive situations do not make decisions based only on a global past record, but also use local information exchange. In other words, we usually acquire information from one another via observation of each others' behavior, and modify our own decisions accordingly. A more realistic model should consider this observation and modification process, and the social networks established among agents.

There are several studies which propose network-based Minority Game models with local information exchange [6], [7]. Remondino and Cappellini introduced a sort of a social network in their model, through which the agents can exchange a tentative statement about their next decision [6]. At the beginning of each step, every agent chooses its alternative (-1) or (1) randomly. Next, each agent collects information about its neighbors' initial choice through the network and then follows the majority side choice of the neighborhood.

In Anghel's model, agents employ a two-step decision making procedure [7]. First, each agent predicts what the minority choice will be based on its own strategy table. However, it does not necessarily act on that prediction. From among its neighboring agents (including itself), it selects the agent which has made the most accurate predictions so far, and adopts its prediction as final choice.

These studies are based on the hypothesis that each agent should follow the majority or a successful neighbor to win the game. However, in the Minority Game, the agent should pick the action opposite to the choice of the majority. Therefore, this hypothesis seems unnatural. So, in contrast, we constructed a new network Minority Game model where agents have a two-step decision making procedure as in previous models, but can modify their initial decision making to pick the minority choice among their neighbors.

The objective of this study is to propose our new model, the *network-based Minority Game with observation and modification processes (NMG)*, and to investigate its fundamental characteristics. We are especially interested in the effect of the out-degree of a directed network between agents (e.g., the number of neighbors) on both the wealth of agents, and global efficiency of the game.

II. Standard Minority Game

First, we will explain the detail of the Standard Minority Game (SMG), initially proposed by Challet and Zhang [2], before our model. In each round of the game, N (odd) agents choose one or the other alternative (0 or 1) independently, and the agents making the minority choice receive a reward. The game is repeated for R rounds, and the agent's wealth is defined as the accumulated reward over all rounds.

Each agent makes its choice h ($\in\{0, 1\}$) based on the common knowledge of the past record. The record indicates the winning choice for the last M rounds, and is represented as a binary sequence of M bits. The value of M represents agents' memory size. The record is generated randomly when the game starts and is updated every round. Each agent has S strategy tables; each of these tables contains all possible 2^M memory states, and the next decision for that state (Table I). Agents decide their choice by checking the common record

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with a strategy table selected from their set of tables.

Strategy tables for each of the agents are generated randomly at the beginning of the game and do not change over the course of the game. Each table has an evaluation score which is updated each round according to its potential success. Each strategy table gets one evaluation point only if it correctly forecasted the minority side, regardless of whether or not it has been used this round. Every round, each agent selects its strategy table with the highest evaluation score (if an agent has two or more strategy tables with the same evaluation score, it randomly selects one of them) and decides its choice on basis of that table. This process can be considered a simple type of reinforcement learning.

TABLE I: STRATEGY TABLE ($M = 3$).

record	decision
{0, 0, 0}	0
{0, 0, 1}	1
{0, 1, 0}	0
{0, 1, 1}	0
{1, 0, 0}	0
{1, 0, 1}	1
{1, 1, 0}	1
{1, 1, 1}	0

III. NMG

As we explained above, in the SMG, each agent makes a decision based only on the common past record without communicating with other agents. However, it is rare for us to make decisions based only on the past in our real society. Decisions are often modified in response to the current actions of others. To express the processes of acquiring information about others' decision making and to modify agents' own decisions in response, we propose a new model, named the network-based Minority Game with observation and modification processes (NMG).

In the NMG, an agent i is represented as a node in a directed network, connected to k_i (odd) other agents via directed links (Fig. 1). We refer to the agents that agent i has directed links to as agent i's neighbors $n_1 \ldots n_{ki}$. Through its links, agent i can acquire a part of the decision making information from its neighbors. The value k is the out-degree of the node, which controls the perception range of an agent. The higher the value of k_i, the more information agent i can acquire from others. The network between agents is connected randomly at the beginning of the game based on the k value of each agent, and it does not change over the course of the game.

Each agent has two decision making mechanisms: *predictor* and *observer*. Agents' final decisions h ($\in$ {0, 1}) is found by taking a weighted sum of the output values from these two mechanisms. At the first stage of the decision making process, each agent predicts the minority choice for this round by applying the predictor procedure to the past record. The predictor has S strategy tables and the same reinforcement learning mechanism as the SMG. This process outputs a tentative choice, h_{pre} ($\in$ {0, 1}) on basis of the common record alone.

Once all agents are finished with the predictor decision making process, each of them runs its observer process. Here agents observe the h_{pre} of their neighbors, and then output value h_{obs} ($\in$ {0, 1}), a tentative decision based on the result of their observations, calculated as follows:

$$h_{obs_i} = Q(h_{pre_{n1}}, h_{pre_{n2}}, \ldots, h_{pre_{nki}}) \quad (1)$$

where the values of $h_{pre_{n1}}, \ldots h_{pre_{nki}}$ are the predictors' results of neighbors $n_1, \ldots n_{ki}$, of agent i, and Q is a function that picks the minority choice from among its argument set. In short, the observer perceives the neighbors' initial predictor decisions and outputs the minority choice among them.

Finally, the agents arrive at their final decision h ($\in$ {0, 1}), by weighing h_{pre} and h_{obs} with weight value w, and summing the resulting values:

$$h_i = g(w_i h_{pre_i} + (1 - w_i) h_{obs_i}) \quad (2)$$

$$g(x) = \begin{cases} 0 \ \ if \ \ x < 0.5 \\ 1 \ \ if \ \ x > 0.5 \\ 0 \ or \ 1 \ randomly \ \ if \ \ x = 0.5 \end{cases}$$

Equation (2) expresses the modification process for final decision. The weights value w is originally an agent-specific parameter. However, in this work, we fix the value of w at 0.5 for all agents. After the final decision is made, the agents that made the minority choice obtain one point as a reward. There is no deduction from the majority agents. The game is repeated for R rounds as the SMG.

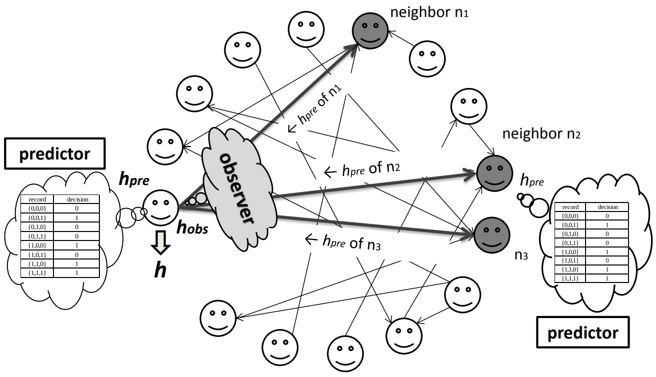

Fig. 1. Concept diagram of the NMG.

IV. RESULTS

A. Perception Range and Agents' Wealth

Wealth and inequality in the Minority Game have been studied in previous works [8], [9]. We analyze the relationship between the value of k, the perception range of the agent, and the wealth of agents to investigate the effect of the newly introduced observation and modification processes.

We conducted a simulation where the out-degree k of all nodes is set to the same value. The parameters are as follows: $N = 21$, $R = 1000$ and M is fixed at 4 to simplify the analysis. Fig. 2 shows the relationship between the average wealth of the agents and the values of k and S. Averages over 1000 samples. When $k = 0$, the agent makes its decisions using the predictor process only, skipping the observer process.

As Fig. 2 illustrates, the wealth of the agents decreases with the increase of out-degree k. This means that expansion of the perception range of all agents causes their wealth to decline. This tendency was observed with other parameter settings as well.

When the value of k increases, it becomes more probable that agents will have common neighbors. As observers of all agents use the same function to output the minority choice among inputs, overlap in neighbors sets increases the probability that agents' observer processes will output the same choice. This causes a global bias of agents' choice. As a result, the number of agents on the majority side increases, which leads to a reduction of the average wealth. In summary, the global expansion of the agents' perception range leads to the loss of diversity in the information that agents receive and hence in their action choices. In this work, we refer to this phenomenon as the *aggregating effect* (Fig. 3).

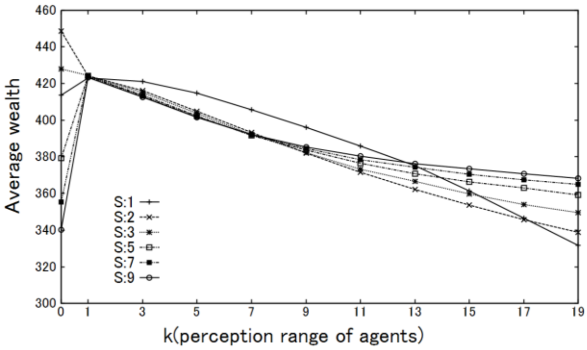

Fig. 2. Average wealth of an agent for varying k and S.

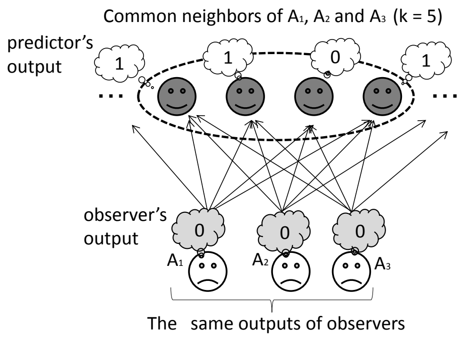

Fig. 3. Concept diagram of the aggregating effect.

Next, we analyzed the relationship between k and the inequality in wealth between agents. Inequalities in wealth can be measured as the standard deviation between agents' wealth. When the standard deviation is high, there is a large wealth inequality between agents. We conducted simulations under the same conditions as mentioned above ($M = 4$). Fig. 4 shows the relation between the values of k, S and the standard deviation of agents' wealth. Averages again over 1000 samples. We see that the inequality increases with the increase of the value of k, especially when $S = 1, 2$. In addition, the slope of the standard deviation curve increases as the value of S decreases. This indicates that the number of strategy tables each individual holds affects global equality.

Wealth inequality also depends on the value of M, that is, the memory capacity of agents. Fig. 5 shows the relationship between values of k, S and global inequality when $M = 10$. This figure shows a decline in inequality in contrast with $M = 4$. It is noteworthy that inequality does not monotonically decrease for $S = 1, 2$, and minimizes at $k = 13, 15$. These simulations demonstrate that the wealth inequality between agents results from the complex interaction between the number of strategy tables, memory size and perception range.

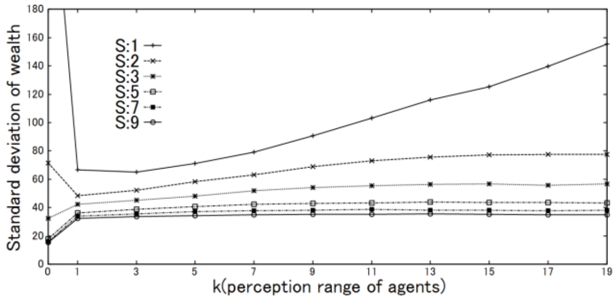

Fig. 4. Standard deviation of agent wealth for each value of k and S ($M = 4$).

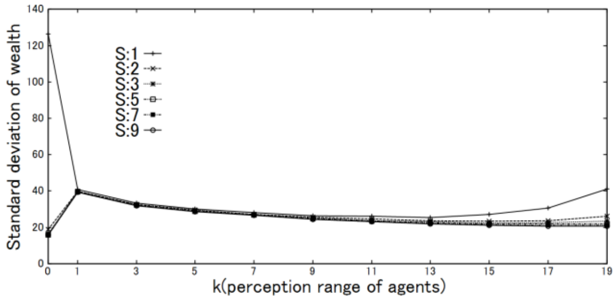

Fig. 5. Standard deviation of agent wealth for each value of k and S ($M = 10$).

B. *Social Efficiency (Volatility)*

In this subsection, we compare the NMG with the SMG for *social efficiency*. Social efficiency (volatility) has been studied in MG related research [10], [11]. It is defined as how many agents acquire the reward by selecting a minority choice, which expresses global efficiency of resource allocation and provides a measure of cooperation among agents. When the number of agents that select a particular choice is close to $N/2$, social efficiency and cooperation are maximized. Therefore, we have after stationary state of $A(t')$, where $A(t')$ is the number of agents which select choice 0 at round t'. Social efficiency is measured by the deviation σ. The lower the value of σ is, the higher the social efficiency is. Economic interpreted, σ could describe the stability of financial markets.

$$\sigma^2 = \lim_{t\to\infty} \frac{1}{t}\int_{t_0}^{t_0+t}\left(A(t') - \frac{N}{2}\right)^2 dt' \qquad (3)$$

Fig. 6 compares social efficiency between the SMG and the NMG when all agents' k is fixed at 1 (we only show σ = 2 - 15). We set $N = 101$ and calculated the average over 100 runs for each combination of S and M. The *random* line shows the value of σ when all agents make their choice at random.

Studies of the SMG have demonstrated that social efficiency is influenced by the values of S and M [10]. However, in the NMG (with $k = 1$) social efficiency is not influenced by either value, and is higher (σ is lower) than with random choice at all values of M. Efficiency is also higher than in the SMG when $M < 5$ and $9 < M$. Fig. 7 shows the same comparison, under the same conditions as elaborated above, except now with parameter k fixed at 3. The figure shows that social efficiency in the NMG is improved when $M < 4$ and $10 < M$ compared to the SMG. Other experiments showed that this improvement weakens as the value of k increases.

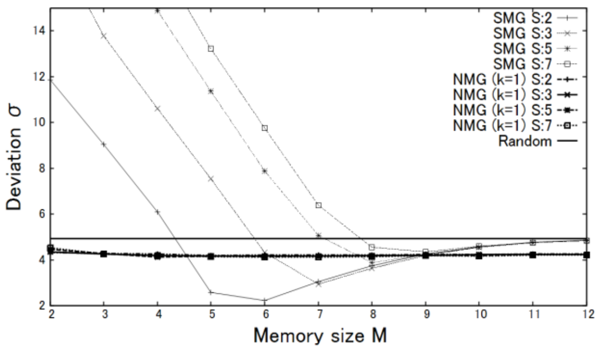

Fig. 6. Comparison of social efficiency between the NMG (k = 1) (bold lines) and the SMG (thin lines).

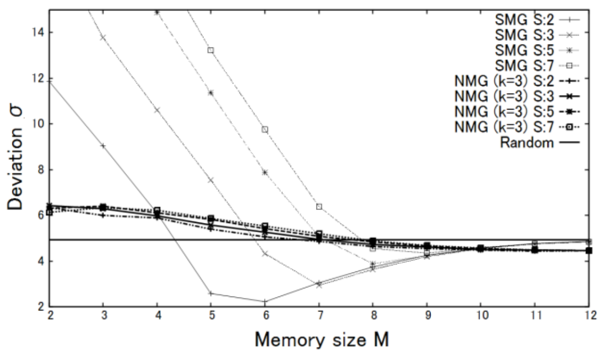

Fig. 7. Comparison of social efficiency between the NMG (k = 3) (bold lines) and the SMG (thin lines).

V. Discussions

One conclusion that can be drawn from the analysis in subsection A of the previous section is that the increase of the perception range of all agents leads to a loss of diversity in the information that different individuals receive, resulting in a reduction of individual wealth. As Akaishi and Arita [12] discussed, loss of diversity resulting from improved perception is a universal problem in decision making in a group. Our model succeeds in expressing this phenomenon. Recently, the improvement of SNS (social networking service), such as Twitter and Facebook, has extended the perception of an individual to larger number of other individuals. However, our results suggest that extension of perception does not always lead to benefit.

In addition, it was found that wealth inequality increases and decreases as we simply expand agents' perception ranges, even if all agents use one and the same perception range. Moreover, we discovered that inequality is affected by the number of strategy tables each agent holds, as well as by their memory size. The values of S (the number of strategy tables that each agent holds) and M (memory size of each agent) can be interpreted as learning ability and recall ability of agents, respectively. The results of our simulations imply that expansion of the perception range expands the wealth gap in a group when individuals have bad memory, and reduce it when they have good memory.

The analysis in subsection B of the previous section also shows an improvement of social efficiency under some conditions compared to the original model (SMG), when the perception range is set small. This indicates that we can enhance cooperation in collective behavior by adding mutual observation and modification mechanisms to the system, that work to retain diversity of action choice.

VI. Conclusion and Future Work

We proposed a new Minority Game variant as an abstract model of socially informed behavior in competition over a resource, and conducted multi-agent simulations to understand its fundamental characteristics. Given our results, we believe that the proposed model provides a useful and interesting platform for investigating the relationship between local information exchange and the global characteristics of a society.

In future work, we are planning to further analyze the mechanism of the emergence of wealth inequality between agents, and look at inequality using the Gini coefficient instead of the standard deviation as the index of inequality.

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